BIMAGIC LABELLINGIN GRAPH

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ABSTRACT

Now a days most of the researchers are doing lots of work in BimagicLabelling of graphs. The purpose of this paper is to introduce new labelling called edge magic labelling and is to obtain the existence of this labelling for certain graphs. An edge magic total labelling of a graph G(V,E) with p vertices and q edges is a bijection f: V (G) U E(G)→ {1, 2,.., p+q} such that f(u)+f(v)+ f(uv) is a constant k for any edge uv∈E(G). If there exists two constants k1 and k2 such that f(u)+f(v) + f(uv) is either k1 or k2, it is said to be an edge bimagic total labelling. A total edge magic (bimagic) graph is called a super edge magic (bimagic) if f(V (G)) = {1, 2,.., p} and it is called super edge magic(bimagic) if f(E(G)) = {1, 2,.., q}. In this paper, we investigate and exhibit super edge magic and bimagiclabelling for some interesting families of graphs.

Keywords: Graph labelling, edge magic labelling

1.INTRODUCTION

A labelling of a graph G is an assignment f of labels to either the vertices or the edges or both subject to certain conditions.Labeled graphs are becoming an increasingly useful family of mathematical models from a broad range of applications. Graphlabelling was first introduced in the late 1960's.A useful survey on graph labelling by J.A Gallian (2010)can be found in[1].All graphs considered are finite , simple and undirected.We follow the notation and terminology of [2]. In most applications labels are +ve integers, though in general real numbers could be used.A (p, q)-graph with p vertices and q edges is called total edge magic if there is a bijection f:VUE→ {1,2,……,p+q} such that there exists a constant k for any edge uv in E with f(u)+f(v)+f(uv)=k. The original concept of total edge-magic graph is due to Kotzig and Rosa [3]. They called it magic graph. A total edge-magic graph is called a super edge-magic if f(V(G))={1,2,……,p}. Wallis [4] called super edge-magic as strongly edge- magic. An Edge antimagic total labelling of a graph with p vertices and q edges is a bijection from the set of edges to 1, 2,……,p+q such that the sums of the label of the edge and incident vertices are pairwise distinct. It becomes interesting when we arrive with magic type labelling summing to exactly two distinct constants say k1 or k2. Edge bimagic totally labelling was introduced by J. Baskar Babujee [5] and studied in [6] as (1,1) edgebimagiclabelling. A graph G(p,q) with p vertices and q edges is called total edge bimagic if there exists a bi-jection f:V∪E→ {1,2,…….p+q} such that for any edge uv∈E we have two constants k1 and k2.

2. SUPER EDGE-MAGIC LABELLING

Definition 2.1.

A graph G with p vertices and q edges is called total edge magic if there is a bijection $f : V \cup E \rightarrow \{1, 2, .., p+q\}$ such that there exists a constant k for any edge uv in E, $f(u) + f(uv) + f(v) = k$. A total edge magic graph is called super edge magic if $f(V) = \{1,2,\ldots,p\}$.

Example 1:

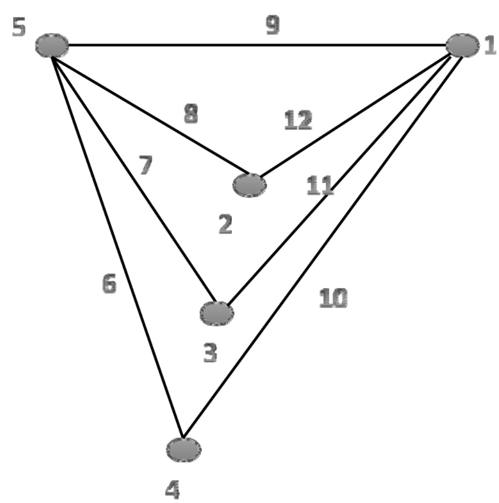

Figure 1: *Super edge magic graph for P_2+3K_1 and magic constant $k = 15$*

Definition 2.2.

A graph G with p vertices and q edges is called total edge bimagic if there exists a bijective function $f: V\cup E \rightarrow \{1,2,\ldots,p+q\}$ such that for any edge uv∈E, we have two constants k_1 and k_2 with $f(u)+f(v)+f(uv) = k_1$ or k_2. A total edge-bimagic graph is called super edge-bimagic if f(V) = {1,2,…,p} .

Definition 2.3.

A vertex switching Gv of a graph G is obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

3. Main Results on Super Edge BimagicLabelling

Theorem 3.1

Switching a pendant vertex in path graph P_n(n ≥ 6) admits super edge bimagiclabelling.

Proof.

Let v_1, v_2, $v3$, …, v_n be the vertices of P_n and G_v be the graph obtained by switching a pendant vertex v of G = P_n . We denote that the pendant vertex v_1 is swiched in G. Then the vertex set

V={v_i ;1≤ i ≤ n} and edge set E = $E_1 \cup E_2$ where E_1=

{$v_{i}\, v_{i+1}$;2 ≤i ≤ n−1}and E_2= {$v_1\, v_i$; 3 ≤ i ≤n} . We note that |V(G)| = n and |E(G)| = 2n-4. We define a bijective function f : V∪ E →{1, 2, ..., 3n-4} is given below.

(i) If n is even

For i = 3 to n-1 :$i \equiv 1(mod\ 2)$, $f(v_i) = \frac{n}{2} + \frac{i+1}{2} - 1$.

For i = 2 to n :$i \equiv 0(mod\ 2)$, $f(v_i) = \frac{i}{2}$.

For i = 3 to n-1 :$i \equiv 1(mod\ 2)$, $f(v_1v_i) = n + \frac{n}{2} - \frac{i+1}{2} + 1$.

For i = 4 to n :$i \equiv 0(mod\ 2)$, $f(v_1v_i) = 2n - \frac{i}{2}$.

$f(v_1) = n$, For i = 2 to n-1 :$f(v_iv_{i+1}) = 3n - 2 - i$.

In the following cases, it is justified that the above assignment results in the required labelling.

case(i) : For any edge $v_i\, v_{i+1} \in E_1$

subcase(i) : $i \equiv 1(mod\ 2)$

$$f(v_i) + f(v_{i+1}) + f(v_iv_{i+1}) = \frac{n}{2} + \frac{i+1}{2} - 1 + \frac{i+1}{2} + 3n - 2 - i = \frac{7n-4}{2} = k_1 .$$

subcase(ii): $i \equiv 0(mod\ 2)$

$$f(v_i) + f(v_{i+1}) + f(v_iv_{i+1}) = \frac{i}{2} + \frac{n}{2} - 1 + \frac{i+2}{2} - 1 + 3n - 2 - i = \frac{7n-4}{2} = k_1.$$

case(ii) : For any edge $v_1\, v_i \in E_2$

subcase(i) : i $\equiv 1(mod\ 2)$

$f(v_1) + f(v_i) + f(v_1 v_i) = n + \frac{n}{2} + \frac{i+1}{2} - 1 + 1 + n + \frac{n}{2} - \frac{i+1}{2} + 1 = 3n = k_2$.

subcase(ii): i $\equiv 0(mod\ 2)$

$f(v_1) + f(v_i) + f(v_1 v_i) = n + \frac{i}{2} + 2n - \frac{i}{2} = 3n = k_2$.

(ii) If n is odd

For i = 3 to n :$i \equiv 1(mod\ 2), f(v_i) = \frac{n+1}{2} + \frac{i+1}{2} - 2.$

For i = 2 to n-1 :$i \equiv 0(mod\ 2), f(v_i) = \frac{i}{2}.$

For i = 3 to n :$i \equiv 1(mod\ 2), f(v_1 v_i) = n + \frac{n+1}{2} - \frac{i+1}{2}+1.$

For i = 4 to n-1 :$i\ 0(mod\ 2), f(v_1 v_i) = 2n - \frac{i}{2}.$

For i = 2 to n-1 :$f(v_i v_{i+1}) = 3n - 2 - i. f(v_1) = n.$

In the following cases, it is justified that the above assignment results in the required labelling.

case(i) : For any edge $v_i\, v(i+1) \in E_1$

subcase(i) : $i \equiv 1(mod\ 2)$

$f(v_i) + f(v_{i+1}) + f(v_i v_{i+1}) = \frac{n+1}{2} - 2 + \frac{i+1}{2} + \frac{i+1}{2} + 3n - 2 - i = \frac{7n-5}{2} = k_1.$

subcase(ii) : $i \equiv 0\ (mod\ 2)$

$f(v_i) + f(v_{i+1}) + f(v_i v_{i+1}) = \frac{i}{2} + \frac{n+1}{2} - 2 + \frac{i+1}{2} + 3n - 2 - i = \frac{7n-5}{2} = k_1.$

case(ii) : For any edge $v_1\, v_i \in E_2$

subcase(i) : $i \equiv 1(mod\ 2)$

$f(v_1) + f(v_i) + f(v_1 v_i) = n + \frac{n+1}{2} - 2 + \frac{i+1}{2} + n + \frac{n+1}{2} + 1 - \frac{i+1}{2} = 3n = k_2$.

subcase(ii): $i \equiv 0(mod\ 2)$

$f(v_1) + f(v_i) + f(v_1 v_i) = n + \frac{i}{2} + 2n - \frac{i}{2} = 3n = k_2$.

From the above cases we have two constants. When n is odd then the constants are $k_1 = \frac{7n-5}{2}$ and $k_2 = 3n$.

When n is even then the constants are $k_1 = \frac{7n-4}{2}$ and $k_2 = 3n$. Hence the graph obtained by switching a pendant vertex in path P_n

admits super edge bimagic labelling.

Example 2.

Taking the graph by switch a pendant vertex in path P_6 admits super edge bimagiclabelling with two constant k_1 and k_2 are given in Figure 2.

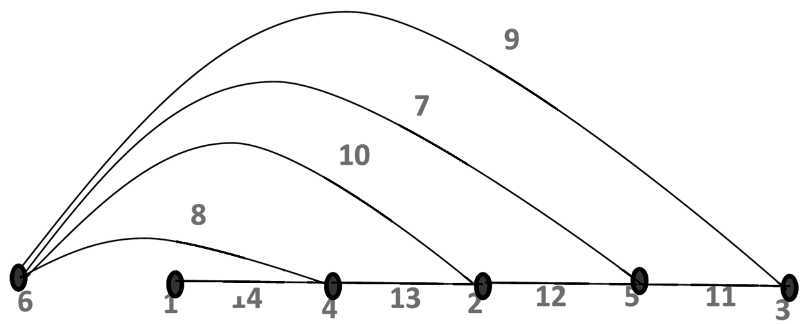

Figure 2: k_1 =18, k_2 = 19

Remark 1. Switching a pendant vertex in pathP_n (n = 3, 4 or 5) admits super edge magic labelling.

Theorem 3.2.

Switching a vertex in cycle graph C_n (n ≥6) admits super edge bimagiclabelling.

Remark 2. Switching a vertex in cycle graph C_n (n =4 or 5) admits super edge magic labelling .

Theorem 3.3.

Switching a pendant vertex in star graph $K_{1,\ n}$ (n ≥ 3) admits super edge bimagiclabelling.

Theorem 3.4. Switching a pendant vertex in crown graph $C_n \Theta K_1$ (n≥ 3) admits super edge bimagiclabelling.

Example 3.

switching a pendant vertex in crown graph $C_1 \Theta K_1$ is given in fig. 3. It is super edge bimagic labelling with two common counts k_1 = 7n+1, k_2= 4n+3. It is also indicated in the same figure.

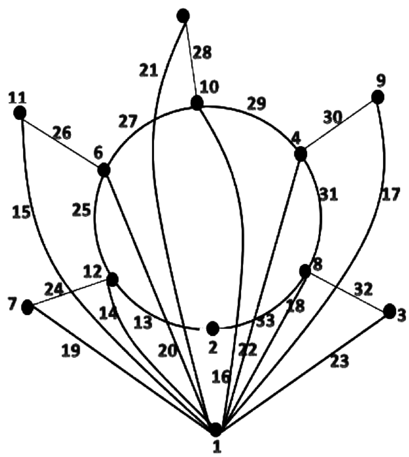

Figure 3: k_1=43, k_2 = 27

Theorem 3.5.

If n is odd then switching the apex vertex in helm graph H_n (n ≥ 3) admits super edge bimagiclabelling.

Proof:

Letv_1, v_2, …, v_{2n+1} be the vertices of H_n and G_v be the graph obtained by switching the apex vertex v of $G = H_n$.

We denote that the apex vertex w is switched in G. Then the vertex set V = $\{v_i ;1 \leq i \leq n\} \cup \{w, u_i ;1 \leq i \leq n\}$ and edge set $E= E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_1 u_n, u_i u_{i+1} ;1 \leq i \leq n-1\}$, $E_2 =\{u_i vi ;2 \leq i \leq n\}$, $E_3 = \{wvi ; 1 \leq i \leq n\}$. We note that |V(G)| = 2n+1 and |E(G)| = 3n. We define a bijective function f : V ∪ E →{1, 2, ..., 5n+1} is given below.

For i = 1 to n ; i ≡ 1(mod 2), $f(u_i) = 2n - \frac{(i+1)}{2}$

For i = 2 to n-1 ; i ≡ 0(mod 2), $f(u_i) = 2n + 2$-½.

For i = 1 to n-1 ; $f(u_i u_{i+1}) = 2n + 1 + i$.

For i = 1 to n ; i ≡ 1(mod 2), $f(u_i v_i) = 4n+2 - \frac{(i+1)}{2}$

For i = 2 to n-1 ; ; i ≡ 0(mod 2), $f(u_i v_i) = 4n - \frac{(n+1)}{2} + 2 - \frac{i}{2}$

For i = 1 to n ; $f(wv_i) = 5n+2-i$.

For i = 1 to n-1 ; $f(v_i) = i+1$. $f(u_1 u_n) = 3n+1$.

In the following cases, it is justified that the above assignment results in the required labelling.

case(i): For any edge $u_i u_{i+1} \in E_1$

subcase(i): i ≡ 1(mod 2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = 2n + 3 - \frac{n+1}{2} - \frac{i+1}{2} + 2n + 2 - \frac{i+1}{2} + 2n + 1 + i$$

$$= \frac{11n+9}{2} = k_1.$$

subcase(ii): i ≡ 0(mod 2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = 2n + 2 - \frac{i}{2} + 2n + 3 - \frac{n+1}{2} - \frac{i+1}{2} + 2n + 1 + i = \frac{11n+9}{2}$$

$$= k_1.$$

subcase(iii): For edge $u_1u_n \in E_1$

$$f(u_1) + f(u_n) + f(u_1u_n) = 2n - \frac{n+1}{2} + 2 + n + 2 + 3n + 1 \frac{11n+9}{2} = k_1.$$

case(ii): For any edge $u_i v_i \in E_2$

subcase(i): i ≡ 1(mod 2)

$$f(u_i) + f(v_i) + f(u_iv_i) = 2n + 3 - \frac{n+1}{2} - \frac{i+1}{2} + i + 1 + 4n + 2 - \frac{i+1}{2} = \frac{11n+9}{2} = k_1.$$

Subcase(ii): i ≡ 0(mod 2)

$$f(u_i) + f(v_i) + f(u_iv_i) = 2n + 2 - \frac{i+1}{2} + i + 1 + 4n - \frac{n+1}{2} + 2 - \frac{i}{2} = \frac{11n+9}{2} = k_1.$$

case(iii): For edge $wv_i \in E_3$

$f(w)+f(v_i)+f(wv_i) = 1+i+!+5n+2-i = 5n + 4 = k_2.$

Therefore, We have two constants from above cases there are

$$k_1 = \frac{(11n+9)}{2} \text{ and } k_2 = 5n+4.$$

Hence the graph obtained by switching the helm graph H_n admits super edge bimagiclabelling.

Example 4.

Switching the apex vertex in helm graph H_7 is given in fig. 4. It is super edge bimagiclabelling with two common counts $k_1 = \frac{11n+9}{2}$ and $k_2 = 5n+ 4$ are also indicated in the same figure.

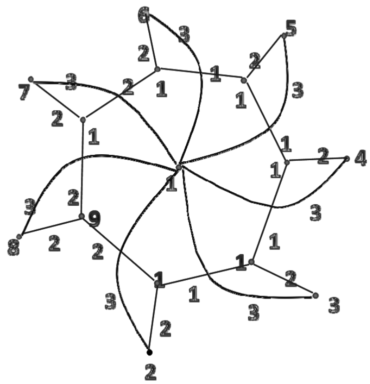

Figure 4: $k_1 = 43$, $k_2 = 39$

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